Period 4, Nov 8, 2024

Identify the polynomial as a $\underbrace{\text{monomial}}_{\text{1 Term}}$, a $\underbrace{\text{binomial}}_{\text{2 Terms}}$, or a $\underbrace{\text{trinomial}}_{\text{3 Terms}}$. Give the degree of the polynomial.

$x^4 + 4x$

Is the polynomial a monomial, binomial, or trinomial?

- ○ monomial
- ○ trinomial
- ● binomial

What is the degree of the polynomial?

$\boxed{4}$

Factor completely.

$15x^4 - 115x^3 + 70x^2 = 5\cdot 3x^4 - 5\cdot 23x^3 + 5\cdot 14x^2 = 5x^2(3x^2 - 23x + 14)$

$= 5x^2(x-7)(3x-2)$

$\underbrace{3x^2 - 23x + 14}_{3\cdot 14 = 42}$ $\Rightarrow$ $\underbrace{3x^2 - 21x}\ \underbrace{-2x + 14}$

$-21\ \ -2$

$3x(x-7) - 2(x-7)$

$(x-7)(3x-2)$

Evaluate the function at the given values.

$h(x) = x^2 - 2x + 4$

a. $h(3)$ **b.** $h(-3)$ **c.** $h(0)$

$h(3) = 3^2 - 2(3) + 4 = 9 - 6 + 4 = 7$

$h(-3) = (-3)^2 - 2(-3) + 4 = 9 + 6 + 4 = 19$

$h(0) = 0^2 - 2(0) + 4 = 4$

The graph of a polynomial function of degree n has, at most, $n-1$ turning points.

The graph of a polynomial function of degree n has, at most, ▼ turning points.

Determine whether the function is a polynomial function. If it is, identify the degree.

$h(x) = 8x^4 + 2x^2 + \frac{3}{x} = 8x^4 + 2x^2 + 3x^{-1}$ None

Determine if the following function is a polynomial function. If it is, identify the degree.

$f(x) = \frac{x^2 - 4}{x^2} = \frac{x^2}{x^2} - \frac{4}{x^2} = 1 - 4x^{-2}$

End behavior

what happens as $x \to \infty$ Right

$x \to -\infty$ Left

$F(x) = -2x^4 + 3x^2 - 5$

degree = 4 = Even Number ⇒ End behavior is Same

$F(-1000) = -2,000,000,000,000 - \cancel{3,000,000,000} - 5$

$F(1000) = -2,000,000,000,000 + \cancel{3,000,000,000 - 5}$

as X Rises y Falls "Fall" Really Negative

X Falls y Falls "Rise" Really Positive

Use the Leading Coefficient Test to determine the end behavior of the graph of the given polynomial function. Then use this end behavior to match the polynomial function with its graph.

degree is odd
End Behavior is opposite

degree=2

degree =3

15. $f(x) = -x^4 + x^2$

X-Rises
Y-Falls
X-Falls
Y-Falls

16. $f(x) = x^3 - 3x^2$

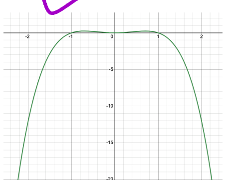
X-Rises
Y-Rises
X → Falls
Y → Falls
different

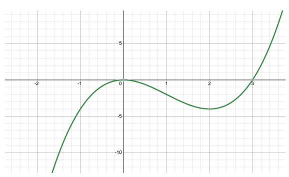
17. $f(x) = (x-4)^2$

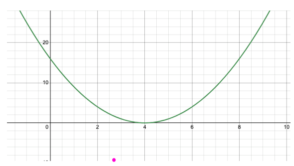
X Rises
Y Rises
X Falls
Y Rises

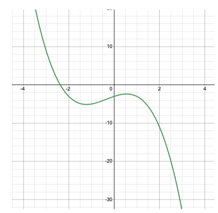
18. $f(x) = -x^3 - x^2 + 2x - 3$

X Rises
Y Falls
X Falls
Y Rise









Use the leading coefficient test to determine the end behavior of the graph of the given polynomial function.

$f(x) = 2x^7 + 3x^6 + 2x^5 + 7$

degree = 7 = odd

different

+2

X Rises

Y Rises

X Falls

Y Falls

Find the zeros for the polynomial function and give the multiplicity for each zero. State whether the graph crosses the x-axis or touches the x-axis and turns around at each zero.

$f(x) = -3(x-8)(x+1)^2$

Graph

$F(x) = -3(x-8)^1(x+1)^2$ = degree = 3 = odd

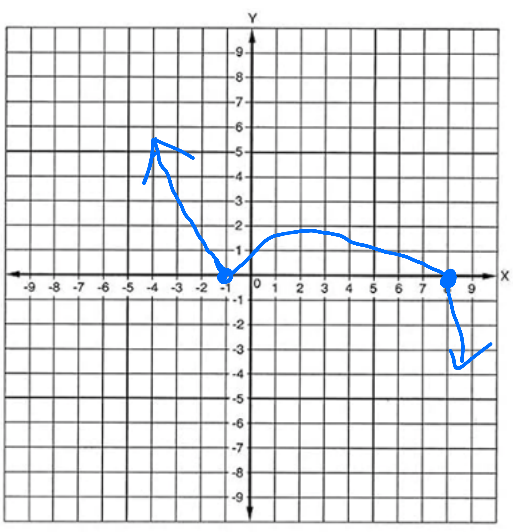
Zeros = 8, −1

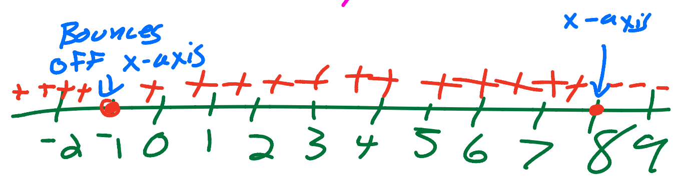
End behavior different

−3

X-Rises → Y Falls

X → Falls → Y-Rises

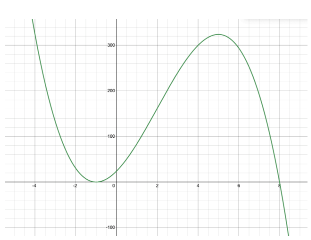




$F(-2) = -3(-10)(-1)^2 = 30 \cdot 1 = 30$

$F(0) = -3(-8)(1)^2 = 24 \cdot 1 = 24$

$F(9) = -3(1)(10)^2 = -3 \cdot 100 = -300$



Other than a no solution set, use interval notation to express the solution set and then graph the solution set on a number line.

$$18\left(\frac{x-18}{6}\right) \geq \left(\frac{x-9}{9} + \frac{5}{18}\right)18$$

$3(x-18) \geq 2(x-9)+5$

$3x-54 \geq 2x-18+5$

$-2x+54 \quad -2x \quad +54$

$x \geq 41$

iF Function goes From + To − or − To +, Had To cross x-axis

Use the Intermediate Value Theorem to show that the polynomial $f(x) = 3x^4 - 6x^2 + 2$ has a real zero between -1 and 0.

$F(0) = 3(0)^4 - 6(0)^2 + 2 = 2$

$F(1) = 3(1)^4 - 6(1)^2 + 2 = -1$

cross x-axis

There is a zero

For the polynomial function $f(x) = x^4 - 10x^3 + 25x^2$, answer the parts **a** through **e**.

No
y-axis symm

$F(-x) = (-x)^4 - 10(-x)^3 + 25(-x)^2 \neq -F(x) = -x^4 + 10x^3 - 25x^2$

$x^4 + 10x^3 + 25x^2$

No origin symm

a. Use the Leading Coefficient Test to determine the graph's end behavior.

y-Rises both ways

b. Find the x-intercept(s). State whether the graph crosses the x-axis, or touches the x-axis and turns around, at each intercept.

Set $y=0$

$0 = x^4 - 10x^3 + 25x^2 = x^2(x^2 - 10x + 25)$

$= x^2(x-5)^2$

Zeros $x = 0, 5$

At which x-intercept(s) does the graph touch the x-axis and turn around? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

Even bounce   Odd cross

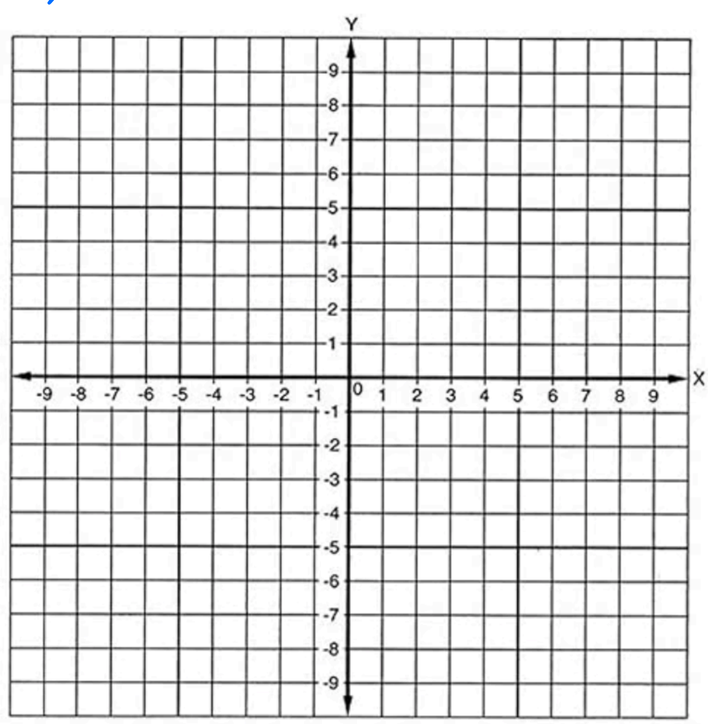
c. Find the y-intercept.

$F(-x) = -F(x)$

d. Determine whether the graph has y-axis symmetry, origin symmetry, or neither. Choose the correct answer below.

$F(-x) = F(x)$

Graph (basic)



Use the given function to complete parts a) through e) below.

cross at $x=3$

$x=-3$

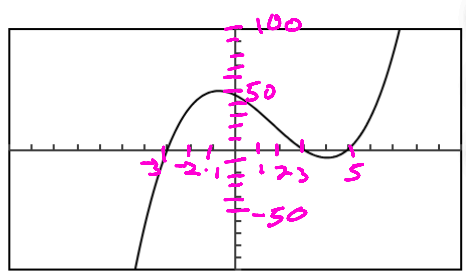
$$f(x) = -3(x-2)^2\left(x^2-9\right) = -3(x-2)^2(x-3)(x+3)$$

bounces

$x=2$

a) Use the Leading Coefficient Test to determine the graph's end behavior.

The graph to the right is a complete graph, that is, it is continuous and displays the function's end behavior. All zeros are integers. Answer the following questions.



[− 10,10,1] by [− 100,100,10]

For the functions $f(x) = 6 - x^2$ and $g(x) = x^2 + 6x - 40$, find $f + g$, $f - g$, fg, and $\frac{f}{g}$. Determine the domain for each function.

$$f(x) + g(x) = 6 - \cancel{x^2} + \cancel{x^2} + 6x - 40 = 6x - 34$$

$$f(x) - g(x) = 6 - x^2 - (x^2 + 6x - 40) = 6 - x^2 - x^2 - 6x + 40 = -2x^2 - 6x + 46$$

$$f(x) \cdot g(x) = (6 - x^2)(x^2 + 6x - 40) = 6x^2 + 36x - 240 - x^4 - 6x^3 + 40x^2$$

$$\frac{f(x)}{g(x)} = \frac{6 - x^2}{x^2 + 6x - 40} = \frac{6 - x^2}{(x+10)(x-4)} \quad \text{Domain } x \neq -10, 4$$

$g(2) = 2 + 7 = 9$

$f(2) = \sqrt{2}$

For $f(x) = \sqrt{x}$ and $g(x) = x + 7$, find the following functions.

a. $(f \circ g)(x)$; **b.** $(g \circ f)(x)$; **c.** $(f \circ g)(2)$; **d.** $(g \circ f)(2)$

$$(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} = \sqrt{x + 7}$$

$$(g \circ f)(x) = g(f(x)) = f(x) + 7 = \sqrt{x} + 7$$

$$(f \circ g)(2) = f(g(2)) = f(9) = \sqrt{9} = 3$$

$$(g \circ f)(2) = g(f(2)) = g(\sqrt{2}) = \sqrt{2} + 7$$

$(x+h)^2 = x^2 + 2xh + h^2$

Find and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$ for the given function.

$f(x) = -x^2 + 4x + 6$

$$f(x+h) = -(x+h)^2 + 4(x+h) + 6$$

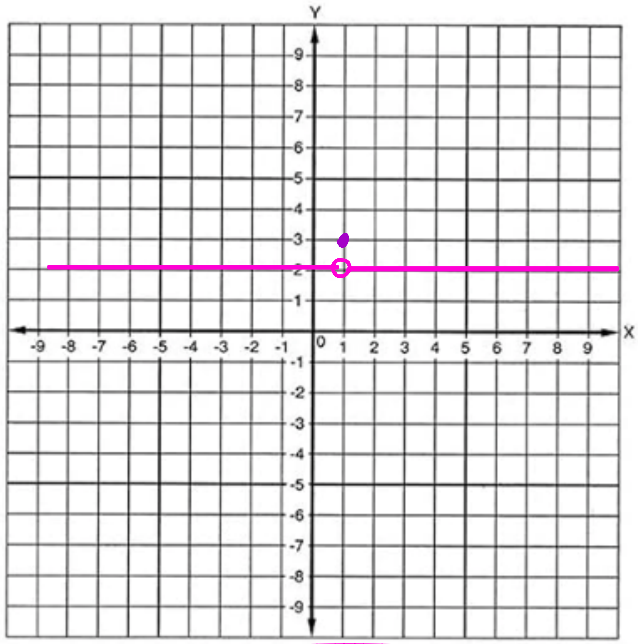
$$\frac{-(x+h)^2 + 4(x+h) + 6 - [-x^2 + 4x + 6]}{h} = \frac{-(x^2 + 2xh + h^2) + \cancel{4x} + 4h + \cancel{6} + x^2 - \cancel{4x} - \cancel{6}}{h}$$

$$\frac{-\cancel{x^2} - 2xh - h^2 + 4h + \cancel{x^2}}{h}$$

$$\frac{\cancel{h}(-2x - h + 4)}{\cancel{h}} = -2x - h + 4$$

Determine whether the following statement makes sense or does not make sense, and explain your reasoning.

I graphed $f(x) = \begin{cases} 2 & \text{if } x \neq 1 \\ 3 & \text{if } x = 1 \end{cases}$, and one piece of my graph is a single point.



You invested $28,000 in two accounts paying 5% and 9% annual interest, respectively. If the total interest earned for the year was $2400, how much was invested at each rate?

X = amount invested at 5%

Y = amount invested at 9%

Total interest = $0.05x + 0.09y$

$x + y = 28000$

$x = \boxed{28000 - y}$

$TI = 0.05(28000 - y) + 0.09y$

$2400 = 1400 - 0.05y + 0.09y$

$-1400 \quad -1400$

$\frac{1000}{0.04} = \frac{0.04y}{0.04} \Rightarrow 25{,}000 = y$

$x = 28000 - y$

$x = 3{,}000$

Find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$ for the given function.

$f(x) = \frac{4}{x}$

$F(x+h) = \frac{4}{x+h}$

$$\frac{\frac{x \cdot 4}{x \cdot (x+h)} - \frac{4(x+h)}{x(x+h)}}{h} = \frac{\frac{4x}{x(x+h)} - \frac{4x+4h}{x(x+h)}}{h} = \frac{\frac{\cancel{4x}-\cancel{4x}-4h}{x(x+h)}}{\frac{h}{1}}$$

$$= \frac{-4\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \frac{-4}{x(x+h)}$$

Consider the function $f(x) = 3x^2 - 30x - 9$.

a. Determine, without graphing, whether the function has a minimum value or a maximum value.
b. Find the minimum or maximum value and determine where it occurs.
c. Identify the function's domain and its range.

$F(x) = \pm a(x-h)^2 + k$
Vertex (h, k)

Complete The Square

$F(x) = 3x^2 - 30x - 9$
$a = 3$

$F(x) = 3(x^2 - 10x + 25 - 25) - 9 = 3[(x-5)^2 - 25] - 9$

$a = 1$
$b = -10$
$\frac{b}{2} = \frac{-10}{2} = -5$

$\left(\frac{b}{2}\right)^2 = (-5)^2 = 25$

$F(x) = 3(x-5)^2 - 75 - 9$
$F(x) = 3(x-5)^2 - 84$
Opens Up
Vertex $(5, -84)$
Domain $\mathbb{R}$
Range $[-84, \infty)$

$-\frac{b}{2a} = \frac{-(-30)}{2(3)} = \frac{30}{6} = 5$

X value of Vertex $x = 5$

$(5, ?)$

$F(5) = 3(5)^2 - 30 \cdot 5 - 9$
$75 - 150 - 9 = -84$
Vertex $(5, -84)$

$F(x) = \pm a(x-5)^2 - 84$

Plug in $x=3$ To $F(x)$
$F(3) = 3(3)^2 - 30(3) - 9$
$27 - 90 - 9 = -72$
$(3, -72)$

$F(3) = a(3-5)^2 - 84 = -72$

Use factoring to solve the following quadratic equation. Check by substitution or by using a graphing utility and identifying x-intercepts.

$36x(x+1) = 187$

$36x^2 + 36x = 187$

$-187 \quad -187$

$36x^2 + 36x - 187 = 0$

$36(\underbrace{x^2 + x}_{} + \frac{1}{4} - \frac{1}{4}) - 187 = 0 \Rightarrow 36\left[(x+\frac{1}{2})^2 - \frac{1}{4}\right] - 187$

$a = 1$

$b = 1$

$\left(\frac{b}{2}\right) = \frac{1}{2}$

$\left(\frac{b}{2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$36(x+\frac{1}{2})^2 - 9 - 187 = 0$

$\frac{\cancel{36}(x+\frac{1}{2})^2}{\cancel{36}} = \frac{196}{36}$

$\sqrt{(x+\frac{1}{2})^2} = \sqrt{\frac{196}{36}} = \pm\frac{14}{6} = \pm\frac{7}{3}$

$x + \frac{1}{2} = \pm\frac{7}{3} \Rightarrow x = -\frac{1}{2} \pm \frac{7}{3}$

$\underbrace{36x^2 + 36x}_{} - 187 = 0$

$36 \cdot -187 = -6 \cdot 6 \cdot 17 \cdot 11$

$6 \cdot 17 \quad -6 \cdot 11 = 102 - 66 = 36$

187: $17 \cdot 11$

36: $6 \cdot 6$

$\underbrace{36x^2 + 102x}_{} \underbrace{- 66x - 187}_{} = 0$

$6x(6x+17) - 11(6x+17)$

$(6x+17)(6x-11) = 0$

$6x + 17 = 0$ or $6x - 11 = 0$

$6x = -17$ $\quad$ $6x = 11$

$x = \frac{-17}{6}$ or $x = \frac{11}{6}$

Use the vertex and intercepts to sketch the graph of the quadratic function. Give the equation for the parabola's axis of symmetry. Use the graph to determine the function's domain and range.

$f(x) = 4x^2 + 8x - 1$

$F(x) = 4(x^2 + 2x + 1 - 1) - 1 \Rightarrow F(x) = 4[(x+1)^2 - 1] - 1$

$a = 1$

$b = 2$

$\frac{b}{2} = \frac{2}{2} = 1$

$\left(\frac{b}{2}\right)^2 = (1)^2 = 1$

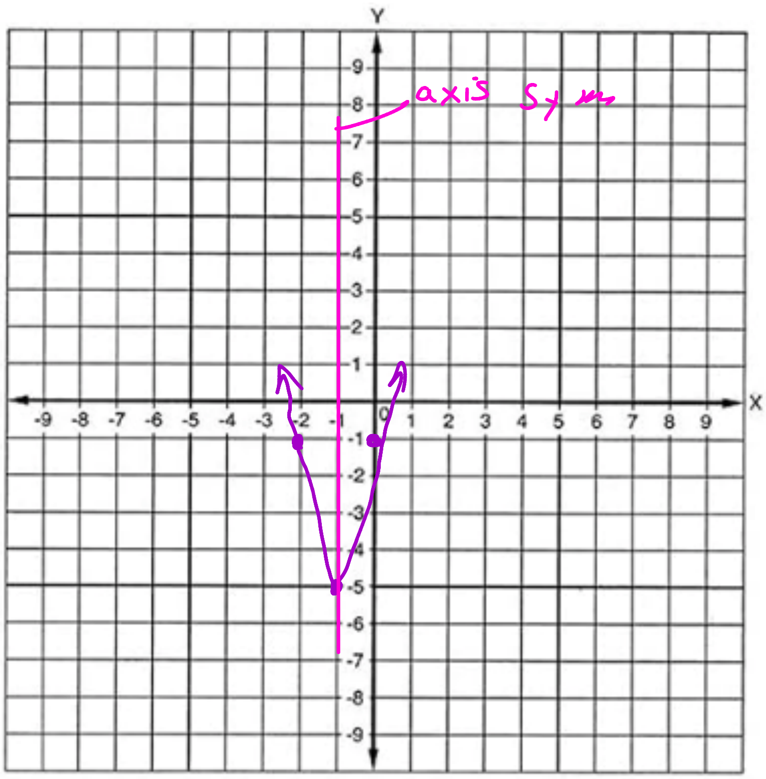
$F(x) = 4(x+1)^2 - 4 - 1$

$F(x) = 4(x+1)^2 - 5$

y-int (set $x = 0$)

$F(0) = 4(0)^2 + 8(0) - 1 = -1$

Vertex $(-1, -5)$ opens up



axis symm

$x = -1$

Domain $\mathbb{R}$

Range $[-5, \infty)$

Write the equation of the following parabola in vertex form.

The vertex is $(\overset{h}{-2}, \overset{k}{-2})$ and the graph passes through the point (0,0).

$F(x) = a(x-h)^2 + k$

$F(x) = a(x-(-2))^2 + -2$

$F(x) = a(x+2)^2 - 2$

$F(0) = 0 = a(0+2)^2 - 2$

$0 = 4a - 2$

$0 = 4a - 2$

$a = \frac{1}{2}$

$F(x) = +\frac{1}{2}(x+2)^2 - 2$

Use the graph to determine the following.

a. the function's domain $= (-\infty, \infty) = \mathbb{R}$
b. the function's range $= (-\infty, -1]$
c. the x-intercepts, if any None
d. the y-intercept, if any $y = -5$
e. the function values f(0) and f(5)
-5 -2

Assume that the graph of the function continues its trend beyond the displayed coordinate grid.